PROBLEM ON 2009 OCTOBER 20

MVHS NUMBER THEORY GROUP

The *square numbers* are numbers which, when viewed as a string of dots, can be placed in the shape of a square. For example the first five are

$1 = 1^2$	Corresponding to a 1 by 1 square
$4 = 2^2$	Corresponding to a 2 by 2 square
$9 = 3^2$	Corresponding to a 3 by 3 square
$16 = 4^2$	Corresponding to a 4 by 4 square
$25 = 5^2$	Corresponding to a 5 by 5 square

Continuing in this fashion, for each integer n there is a square n^2 corresponding to an n by n square. Suppose we do the same geometrical construction, but instead of using a square we use a triangle. The first 3 *triangular numbers* are

$$1, 3, 6, \cdots$$

write down the next 5 along with their consecutive differences. What is the relationship between the first n triangular numbers and

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

the sum of the first n integers? As you may have learned, the sum of the first n integers has an explicit form written above in terms of n. Write the explicit form for the sum of the first n triangular numbers. Can you generalize this to get an explicit form for the sum of the first n square numbers? The problem above is worth **1 Point**.

The triangular numbers share an interesting geometrical relationship with the square numbers. Can you find out what it is? What is the algebraic analogy of this relationship? Which relationship is easier for you to observe? This part is worth **2 Points**.